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Maximizing Pedestrian Flow in Shopping Center Layout

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ABSTRACT

A problem frequently faced by shopping center planners is how to locate the shops in relation to one another. A natural objective is to achieve a suitable level of pedestrian movement throughout the center and a good layout will increase overall profitability. This requires the knowledge of pedestrian flows between pairs of shops, by itself a difficult problem. Given the relevant data is available, the optimal layout problem is a hard combinatorial optimization problem. If the shopping center is linear, it is akin to a large scale seriation problem. We present a dynamic programming algorithm for this case, as well as a greedy heuristic. This heuristic is adapted to the case of tree-shaped shopping centers. It is illustrated on a numerical example.

Keywords: shopping centers, shopping malls, store location, seriation problem.

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RÉSUMÉ

Un problème auquel sont souvent confrontés les planificateurs de centres commerciaux est celui de la localisation des magasins les uns par rapport aux autres. Un objectif naturel consiste à maximiser le flux piétonnier à travers le centre commercial, puisque cela devrait influencer les ventes de façon positive. Pour résoudre ce problème, il est nécessaire de connaître les flux entre toutes les paires de magasins, un problème difficile en soi. Si ces données sont disponibles, la localisation des magasins donne lieu à un problème d'optimisation combinatoire difficile. Dans le cas où le centre commercial possède une structure linéaire, le problème est semblable au problème de sériation. On propose un algorithme de programmation dynamique et une procédure heuristique pour ce cas. On adapte également la procédure heuristique proposée au cas de structures en forme d'arbre. Cette méthode est illustrée à partir d'un exemple numérique.

Mots-clés: centres commerciaux, localisation de magasins, problème de sériation.

1. Introduction

Since the 1950's, shopping centers have been part of our landscape and of our daily life. Their image and their role have changed over the years, as they have evolved from purely trading places to "a local downtown region, a community meeting and activities place, a recreational area, or the place for people to be away from home – where they can shop, eat, be entertained, meet their friends, neighbors, and business contacts, and engage in activities such as community affairs" (Lion, 1976, p. 2). Recently, the Mall of America in Minnesota, the largest enclosed shopping mall in the United States, has even approved a plan to place classrooms among its stores (Hillbery, 1992). The economic importance of shopping centers cannot be overestimated: thus in the United States, they account for more than 55% of all non-automobile retail sales (Monitor, 1990).

The success of a shopping center is highly dependent upon careful planning. According to a case study by Bean, Noon, Ryan and Selton (1988), the development of new centers takes place in three main phases. First, major feasibility factors such as marketing demographics, location, building costs and expected revenues are examined. A tentative floor plan of the center is then established, together with the location of anchor tenants, usually supermarkets or large department stores. These determine the character of the shopping center. Finally, smaller tenants are selected. Several studies have been devoted to a number of aspects of this process, such as location (Davis and Rogers 1987, Wrigley 1991, Moore and Attewell 1991), architectural design (Keyworth 1989, Connor 1990) and tenant mix (Seagle 1967,

Jensen 1980, Bean, Noon, Ryan and Salton 1988). However, relatively little attention has been paid to the important problem of locating stores in relation to one another, though some authors, e.g., Ghosh (1986), have studied the hierarchical organization of retail centers and the pattern of market areas, but not in the specific context of shopping malls. In a recent study, Baron (1991) insisted on the necessity of ensuring a significant pedestrian flow throughout the shopping center so as to avoid dead areas. Lion (1976, p. 34) writes about the need "to balance the traffic flow throughout the center". To a large extent, traffic flow translates into store exposure, sales and ultimately, overall profitability.

In our view, there are two major components to the store location problem. The first is to develop an ability to estimate the traffic pattern resulting from a given configuration of store locations. The second component is the development of a shopping center store layout procedure that will maximize overall pedestrian flow.

Relatively little work has been done in the area of pedestrian flow models. One major problem, it seems, is the difficulty of obtaining reliable traffic counts (Baron 1991). In an earlier study, the authors developed an alternative methodology that can help estimate pedestrian flows in an existing shopping mall, using data on the various subsets of shops patronized by customers (Eiselt, Gendreau and Laporte, 1992). A recent experiment conducted at the Fredericton Shopping Mall in New Brunswick (Canada) confirmed the feasibility of the proposed approach. For two weeks, interviewers positioned at the mall's various exits asked customers in which shops they had made purchases and which door they had used to enter the mall. By

making a number of reasonable behavioral assumptions, pedestrian flows were then estimated using this information.

The object of this paper is not to show how traffic flows can be reconstructed for an existing mall, but rather to develop a location planning model based on the same type of data used in the Fredericton experiment. This can be obtained from surveys conducted at comparable shopping malls, or from shopping intentions surveys. We will implicitly or explicitly be making a number of assumptions. First and most importantly, the proposed methodology can only be used in contexts in which the mall management selects tenants. Such is the case when a shopping center is first designed (Bean, Noon, Ryan and Salton, 1988). Later, as some stores close and new tenants have to be found, the mall planners may not have much choice, even though some of them do actively pursue desirable tenants. Second, we assume that customers are more likely to purchase in a store if they walk past it and therefore that the maximization of pedestrian flow will lead to the maximization of purchases. From Baron's (1991) paper, this looks like a reasonable assumption. Third, our approach only applies to shopping centers that are tree-shaped (as an example, see Figure 1). While this is the case of a large number of small or traditional shopping centers in North-America, layouts for some newer and sometimes very large malls are often more complicated (see, e.g., Carpenter 1974).

The remainder of this paper is organized as follows. In Section 2, we construct a normative model for the case of linear shopping malls. We then show how the location problem on a line can be formulated as a dynamic programming problem (Section 3), and solved by means of a greedy heuristic procedure (Section 4); an extension

of the proposed heuristic to the case of tree-shaped malls is provided. Finally, in Section 5, the greedy procedure is illustrated on an example, using real data. The conclusion follows in Section 6.

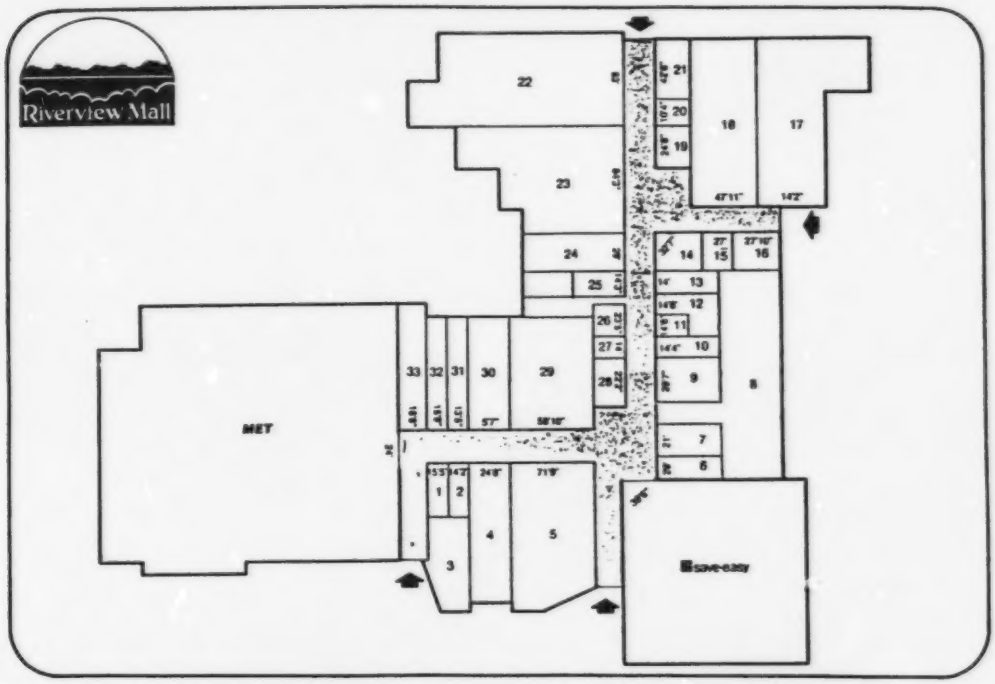


Figure 1. A tree-shaped shopping mall: the Riverview Mall in New Brunswick, Canada

2. Model for the linear case

In this section, we develop an optimization model for the *store location problem* (SLP) on a line: given customers' shopping patterns and their frequencies, determine how n stores should be located on a line so as to maximize the total pedestrian flow between all stores. The relevant input information is summarized by the set of all pairs (P_r, d_r) where P_r is a store *pattern*, i.e., a non-empty subset of the set of all

stores $S = \{1, ..., n\}$, and d_r is the number of customers patronizing the stores of P_r (in the Fredericton survey, this was defined as the number of customers who had effectively made purchases at those stores). The data can be represented by means of a binary matrix A with n rows representing the stores, and p columns representing the observed shopping patterns $P_1, ..., P_p$. A "1" is then entered in row i and column r if and only if pattern P_r includes store i. For example, suppose $S = \{1, 2, 3, 4, 5\}$ and assume the observed shopping patterns and frequencies are as in Table 1:

Table 1. Store patterns and associated frequencies

Store pattern (P_r)	Frequency (d_r)
$P_1 = \{1\}$	8
$P_2 = \{2\}$	4
$P_3 = \{1, 2\}$	3
$P_4 = \{1, 3\}$	5
$P_5 = \{1, 4\}$	2
$P_6 = \{2, 3\}$	3
$P_7 = \{2, 5\}$	4
$P_8 = \{1, 2, 4\}$	3
$P_9 = \{1, 2, 5\}$	5
$P_{10} = \{1, 2, 4, 5\}$	1

The A matrix is given in Figure 2.

The SLP on a line is then to determine a permutation π of the rows of A in order to

$$\text{maximize } f(\pi) = \sum_{r=1}^{p} d_r(\beta_r - \alpha_r), \tag{1}$$

stores \ patterns	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}
1	1		1	1	1			1	1	1
2		1	1			1	1	1	1	1
3				1		1				
4					1			1		1
5							1		1	1
d_r	8	4	3	5	2	3	4	3	5	1

Figure 2. The A matrix corresponding to the 10 pattern example

where α_r and β_r are the row numbers of the first and last "1" respectively in the permuted matrix. The idea behind this objective function is as follows. Suppose that the stores in the mall are arranged along a line and numbered from 1 to n. This way, the leftmost store patronized in a certain shopping pattern has the smallest and the rightmost store has the largest index. Any customer using this shopping pattern will then be exposed to all stores between these two extremes. The assumption is now that if a customer passes by a store and is exposed to its displays, there is a certain probability that he will patronize that store. Thus, the term $\beta_r - \alpha_r$ in (1) denotes the number of stores that a customer who uses the r^{th} shopping pattern is exposed to. This term is assumed to be linearly related to the total purchases a customer makes. Consequently, the weighted sum of these terms is a function of the expected sales which are to be maximized. This model implicitly relies on a number of simplifying assumptions, namely:

- the traffic flow generated by each shopper exactly coincides with the range of shops he patronizes; the model ignores entrances and exits – unlike stores, these cannot

be located at will, but are usually fixed architectural features;

- the model also assumes that even in two-sided linear malls, all stores can be located consecutively on a single line;
- store front lengths are ignored: these could easily be taken into account, however, by defining c_k, the front length of store k, and replacing the $(\beta_r - \alpha_r)$ factor in (1) by $\sum_{k \in C_r} c_k$, where C_r is the set of stores between the rows of the first and last "1" in column r of the permuted A matrix.

This model is in fact similar to the seriation problem (SP) encountered in archaeology and in various fields related to behavioral sciences (see, e.g., Hubert (1974), Shuchat (1984) and Laporte (1987)). The only difference is that in the seriation problem, the objective $f(\pi)$ is to be minimized rather than maximized. Our problem is also related to the one-dimensional space allocation problem (ODSAP) treated by Picard and Queyranne (1981): here, each pair of stores (k, ℓ) generates a traffic flow $d_{k\ell}$. Assume store k is located in position i and store ℓ in position j, and let the distance between i and j be c_{ij}, then the ODSAP is to

$$\text{minimize} \quad g(\pi) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{\ell=i}^{n} c_{ij}\, d_{k\ell} \tag{2}$$

over all assignments π of stores to positions. This problem is a special case of the quadratic assignment problem (QAP) originally formulated in the same way by Koopmans and Beckmann (1957). It can be shown that the SLP, the SP, the ODSAP, and the QAP are all NP-hard in the strong sense, by transformation from the Hamiltonian circuit problem known to NP-complete in the strong sense, see Sahni and

Gonzalez (1976) and Garey and Johnson (1979). For a general overview of location problems with spatial interactions, see Burkard (1990).

3. A Dynamic Programming Formulation for the SLP on a Line

Like the seriation problem, the SLP on a line can be formulated as an integer linear program (Doran and Powell (1972), Laporte (1987)) or as a dynamic program (DP) (Adelson, Laporte and Norman (1976), Laporte (1987)). As shown by Laporte in the case of the SP, the DP formulation is more robust and enables larger problems to be solved. We propose a new dynamic programming formulation for the linear SLP.

In order to formulate the problem, define

$S = \{1, ..., n\}$	:	the set of all stores;
$\mathcal{P} = \{P_1, ..., P_p\}$	:	the set of all p relevant patterns;
d_r	:	the number of customers with pattern P_r;
$S_j^k, j = 1, ..., \binom{n}{k}$	:	the subsets of k stores among n.

The basic idea behind the proposed DP algorithm consists of locating stores sequentially. It is assumed all stores of S_j^k have already been located (in some order in positions $1, ..., k$) and one must determine the best store to locate in position $(k+1)$. To this end, define $f_k^*\left(S_j^k\right)$, the traffic flow in the interval $[k, n]$ if all stores of $S \setminus S_j^k$ are optimally located in positions $k+1, ..., n$. The value of $f_k^*\left(S_j^k\right)$ is then obtained by means of the following recursion formula for $k = 1, ..., n-1$ and $j = 1, ..., \binom{n}{k}$:

$$f_k^*\left(S_j^k\right) = \max_{s \in S \setminus S_j^k} \left[\sum_{P_r \in \mathcal{P}(S_j^k)} d_r + f_{k+1}^*\left(S_j^k \cup \{s\}\right) \right], \tag{3}$$

where $\mathcal{P}\left(S_j^k\right) = \{P_r \in \mathcal{P} \mid \exists s', s'' \in P_r$ such that $s' \in S_j^k$ and $s'' \in S \setminus S_j^k\}$.

Alternatively, the recursion can be written as

$$f_k^*\left(S_j^k\right) = \sum_{P_r \in \mathcal{P}\left(S_j^k\right)} d_r + \max_{s \in S \setminus S_j^k} \left\{ f_{k+1}^*\left(S_j^k \cup \{s\}\right)\right\}. \tag{4}$$

Here, $\sum_{P_r \in \mathcal{P}\left(S_j^k\right)} d_r$ is the total number of customers who will travel between positions k and $k+1$ if all stores of S_j^k occupy the k first positions. In this formulation, the boundary condition is $f_n^*(S) = 0$ and the problem is to determine $f_0^*(\varnothing) = \max_{s \in S} \{f_1^*(\{s\})\}$, together with the optimizing store sequence.

4. A Greedy Heuristic Algorithm

Since the DP formulation leads to an algorithm of exponential time complexity, it can only be applied to relatively small problems. Instead, we propose for larger problems the use of the following greedy algorithm. This procedure uses only part of the information of the A matrix, namely only those columns corresponding to patterns P_r with $1 \leq |P_r| \leq 3$. We believe the loss of information resulting from the use of this restricted set of patterns is very small in practice. For example, in the Fredericton survey, 97% of all customers made purchases at three stores or less (Eiselt, Gendreau and Laporte, 1992).

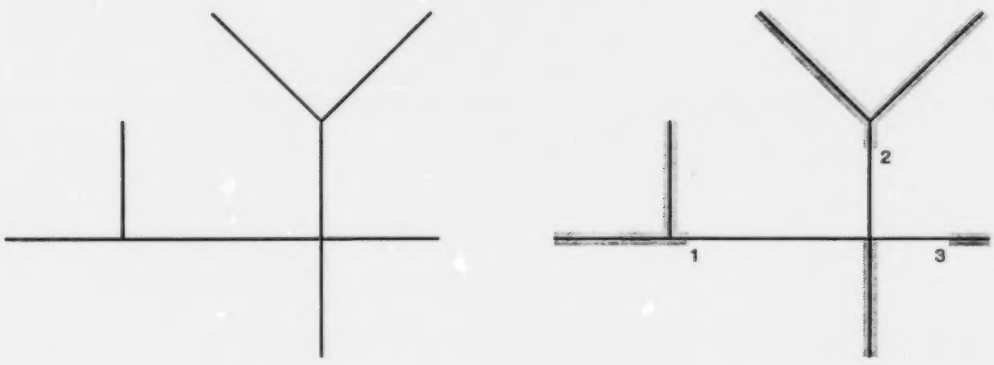
We first define the following *interaction coefficients*:

d_{ij} : the number of customers patronizing stores i and j;

$d_{i,L,\overline{R}}$: the number of customers patronizing store i, at least one store of $L \subseteq S$, and no store of $R \subseteq S$.

The algorithm starts by locating the two stores with the largest interaction d_{ij} at both ends of the line. At a general step of the algorithm, let L and R be the sets of stores located at the left and right extremities. A store i not yet located is then located in the left-most or right-most available position, according to the largest of $d_{i,R,\overline{L}}$ or $d_{i,L,\overline{R}}$, if this maximum is positive. If the maximum is equal to zero, the process is restarted by locating in the two extreme available positions the two stores with the largest interaction, or by locating the last store in the last remaining position.

The algorithm can easily be extended to the case of tree-shaped shopping malls, by gradually "filling up" the tree branches. Throughout the process, the unfilled part of the mall remains a tree, but the number of its branches diminishes gradually. After the initialization step, the remaining tree has a set of "available extremities" corresponding to store locations closest to the empty part of the tree (see Figure 3). When store i is tentatively assigned to a given extremity t_0, distances $c_{t_0 t}$ are needed, in addition to the interaction coefficients: here t_0 is the first available position in a given extremity of the tree, and t is the position of the store last located in any other extremity. Here is a step-by-step description of the algorithm.



a) Initial tree with no store yet assigned

b) Partially filled tree (shaded part) with 3 remaining available extremities

Figure 3. Tree being gradually filled up

ALGORITHM LOCATE-ON-TREE

Step 1 (Initialization). Select two stores i and j maximizing d_{ij} and locate these stores at the two most distant extremities of the tree. Then sequentially allocate one store i to each available extremity t_0 of the tree by selecting each time an unassigned store i maximizing $c_{t_0 t}\, d_{ij}$, where j is a store already located at extremity t.

Step 2 (Termination). If all stores have been located, stop. If only one store is not yet located, assign it to the last available position and stop.

Step 3 (Locating one store). Determine the unassigned store i and the available extremity t_0 maximizing $c_{t_0 t}\, d_{i,R(t),\overline{R}(t_0)}$, where $t \neq t_0$ and $R(t)$ (resp. $R(t_0)$) is the set of stores already located in extremity t (resp. t_0). If the maximum is equal to 0 go to Step 4. Otherwise, locate store i in position t_0 and go to Step 2.

Step 4 (Locating two stores). Determine the two stores i and j with the largest d_{ij}. Assign i and j to the two most distant available extremities. □

Two computational remarks are now in order. First, restricting our attention to patterns P_r with $1 \leq |P_r| \leq 3$ allows for easy updating of the interaction coefficients. Indeed, after placing store i in extremity t_0, the interaction coefficient $d_{j,R(t),\overline{R}(t_0)}$ is updated by subtracting from its current value $d_{j,R(t),\{i\}}$, i.e., the number of customers patronizing i, j and some $r \in R(t)$ (there can be no customers patronizing i, j, $r \in R(t)$ and $r_0 \in R(t_0)$ since $|P_r| \leq 3$); similarly, $d_{j,R(t_0),\overline{R}(t)}$ is updated by adding to its current value the number of customers patronizing i, j, and no store in $R(t_0) \cup R(t)$ (again this is easily accomplished since in any pattern containing i and j there is at most a third store). Branch mergers are also easily handled: when a branch t_0 has been filled up to its merging point with branches $t_1, ..., t_k$, some of which having yet unfilled positions, then extremity t_0 can be disregarded but $R(t_0)$ is kept in memory; when all branches $t_1, ..., t_k$ are filled, these are then merged into a single branch $\bar{t}$ with $R(\bar{t}) = R(t_0) \cup R(t_1) \cup \ ... \ \cup R(t_k)$.

Some extensions and adaptations can be envisaged. For example, the algorithm can easily be modified to account for fixed store locations. This is in fact more often than not the case: most shopping centers are indeed developed around anchor stores located at extremities of the mall. The location problem then only applies to the remaining smaller stores. (Note that in practice, our algorithm will in any case usually locate the larger stores at extremities since they tend to have the largest degree of interaction with other stores.) Also, in some two-storey malls, it is common to preassign some stores to the upper level, typically clothing stores. The SLP can then be decomposed and solved separately for each level.

5. Numerical example

The greedy procedure on a line is now illustrated on a small 10–store example randomly extracted from the Fredericton survey data (see Table 2).

Table 2. List of 10 stores extracted from the Fredericton survey

Store number	Type
1	Women's wear
2	Sports articles
3	Records
4	Toys
5	Men's wear
6	Souvenirs and games
7	Drug mart
8	Coffee shop
9	Fabrics
10	Greeting cards

The following store patterns (P_r) and frequencies (d_r) shown in Table 3 were observed. Since there are 10 stores to locate, the algorithm requires 9 iterations.

At iteration 1, we determine the store pair with the largest interaction coefficient d_{ij}. Here, $\max_i \{d_{ij}\} = d_{7,8} = 6$. Thus, stores 7 and 8 are located at both extremities and the partial arrangement (7,–,–,–,–,–,–,–,–,8) is obtained.

At iteration 2, we compute $\max_i \left\{ d_{i,\{7\},\{\overline{8}\}} \right\} = \max \{d_{1,7}, d_{2,7}, d_{3,7}, d_{4,7}, d_{6,7}, d_{9,7}\}$ $= d_{3,7} = 5$; $\max_i \left\{ d_{i,\{8\},\{\overline{7}\}} \right\} = \max \{d_{2,8}, d_{3,8}, d_{4,8}, d_{5,8}, d_{9,8}, d_{10,8}\} = d_{4,8} = 3$. Since $d_{3,7} > d_{4,8}$, store 3 is located next to 8 and the partial store arrangement is now (7,–,–,–,–,–,–,–,3,8).

Table 3. Selection of observed store patterns and frequencies in the Fredericton survey

P_r	d_r	P_r	d_r	P_r	d_r	P_r	d_r
{1}	14	{1,7}	2	{3,8}	1	{7,9}	2
{2}	20	{1,9}	1	{3,10}	1	{8,9}	1
{3}	39	{1,10}	2	{4,5}	1	{8,10}	2
{4}	37	{2,3}	2	{4,7}	3	{2,3,6}	1
{5}	25	{2,5}	1	{4,8}	3	{2,3,8}	1
{6}	11	{2,6}	1	{4,10}	1	{2,5,8}	1
{7}	152	{2,7}	1	{5,6}	1	{4,8,10}	1
{8}	64	{2,8}	2	{5,7}	1		
{9}	11	{3,6}	1	{6,7}	4		
{10}	22	{3,7}	5	{7,8}	6		

At iteration 3, we find $\max_i \left\{ d_{i,\{7\},\{\overline{3,8}\}} \right\} = d_{6,7} = 4$ and $\max_i \left\{ d_{i,\{\overline{7}\},\{3,8\}} \right\} = d_{4,8} = 3$. Since $d_{3,7} > d_{4,8}$, store 6 is located next to 3 and the partial layout is then (7,–,–,–,–,–,–,6,3,8).

Iterations 4 to 8 are similar: at iterations 4 to 6, stores 4, 1 and 9 are successively located on the right-hand side of the line and, at iterations 7 and 8, stores 10 and 2 are successively located on the left. At iteration 9, only store 5 remains to be located. The final store layout is then π = (7, 10, 2, 5, 9, 1, 4, 6, 3, 8).

The total pedestrian flow can be computed by constructing the A matrix given in Figure 4, corresponding to this ordering (the frequencies d_r are given in the last line). The total pedestrian flow corresponding to this solution is computed as in relation (1):$f(\pi) = \sum_{r=1}^{27} d_r(\beta_r - \alpha_r) = 2 \times (6-1) + \ \ldots \ + 1 \times (10-2) = 266$. Note that only 27 patterns are used since all store patterns consisting of a singleton can be disregarded as they do not contribute to the objective function.

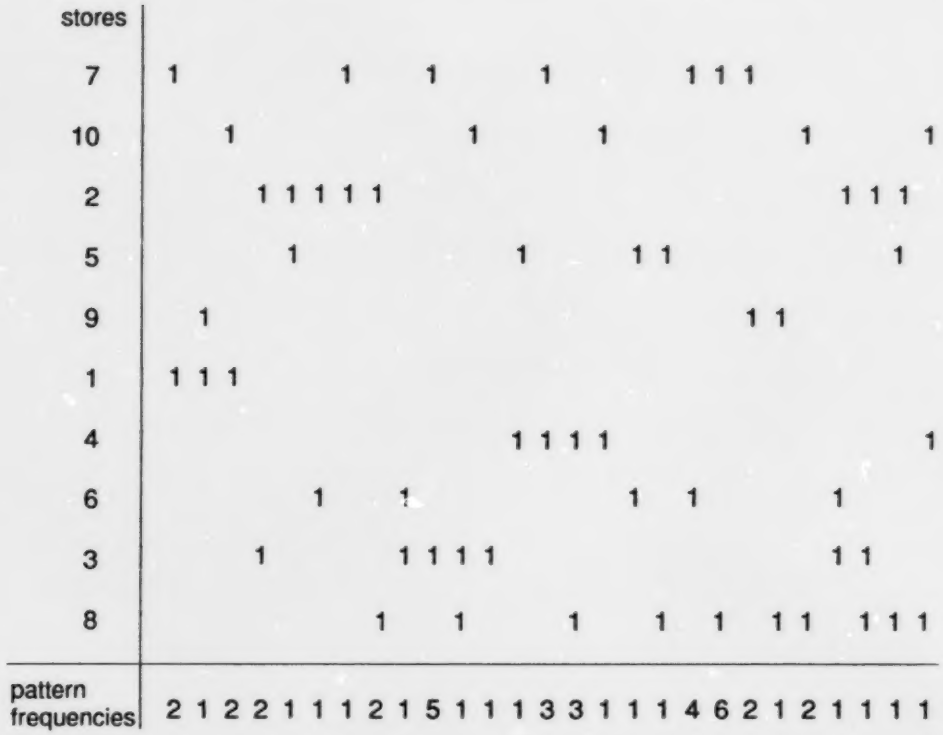


Figure 4. The A matrix corresponding to the optimized 10 store example

6. Conclusion

We have studied in this paper the problem of locating stores in relation to one another in shopping malls in order to maximize pedestrian flow. The choice of this objective is supported in part by the recent literature (see, e.g., Baron 1991), and also by the widespread practice of locating anchor tenants at extremities of corridors. Whether or not an optimized solution can always be implemented is debatable, but there is some value in knowing which layout is likely to generate the largest overall

traffic flow. It is sometimes argued that this objective is socially objectionable as it runs against customers' efficiency. However, the arrangement of stores in most shopping malls, and indeed floor layouts within large supermarkets and department stores, tend to reflect this objective.

A number of assumptions were necessary in order to bring the store location method into a form amenable to standard optimization techniques, and then the problem remains NP-hard even in the simple case of a linear structure. A dynamic programming formulation was proposed for the linear case but it can only work for relatively small problems. The greedy heuristic, on the other hand, is more efficient; it is quite flexible and extends easily to the case of a tree-shaped mall. Both the exact algorithm and the heuristic require shopping pattern forecasts: the Fredericton Mall survey indicates that such patterns can easily be estimated from comparable shopping malls. Finally, the heuristic algorithm only makes use of patterns comprising three stores or less, but it seems that in practice very little information is lost through this simplification.

To the authors' knowledge, this paper is the first documented attempt to address the store location problem. Several extensions are possible. These include the study of malls that are not tree-shaped and the impact of entrance locations on pedestrian traffic. Another line of research is to study how pedestrian flow information can be included in the relative pricing of store locations.

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